CS 188: Artificial Intelligence

Review of Utility, MDPs, RL, Bayes' nets

DISCLAIMER: It is insufficient to simply study these slides, they are merely meant as a quick refresher of the high-level ideas covered. You need to study all materials covered in lecture, section, assignments and projects !

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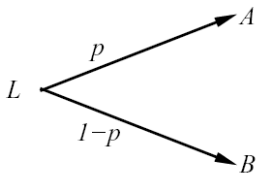
Many slides adapted from Dan Klein

Preferences

- An agent must have preferences among:
  - Prizes: A, B, etc.
  - Lotteries: situations with uncertain prizes



$$L = [p, A;\ (1-p), B]$$

- Notation:

$A \succ B$	A preferred over B
$A \sim B$	indifference between A and B
$A \succeq B$	B not preferred over A

2

Rational Preferences

- Preferences of a rational agent must obey constraints.
  - The axioms of rationality:

Orderability

$$(A \succ B) \vee (B \succ A) \vee (A \sim B)$$

Transitivity

$$(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$$

Continuity

$$A \succ B \succ C \Rightarrow \exists p\ [p, A;\ 1-p, C] \sim B$$

Substitutability

$$A \sim B \Rightarrow [p, A;\ 1-p, C] \sim [p, B; 1-p, C]$$

Monotonicity

$$A \succ B \Rightarrow$$
$$(p \geq q \Leftrightarrow [p, A;\ 1-p, B] \succeq [q, A;\ 1-q, B])$$

- Theorem: Rational preferences imply behavior describable as maximization of expected utility

3

MEU Principle

- Theorem:
  - [Ramsey, 1931; von Neumann & Morgenstern, 1944]
  - Given any preferences satisfying these constraints, there exists a real-valued function U such that:

$$U(A) \geq U(B) \ \Leftrightarrow \ A \succeq B$$

$$U([p_1, S_1;\ \ldots\ ;\ p_n, S_n]) = \sum_i p_i U(S_i)$$

- Maximum expected utility (MEU) principle:
  - Choose the action that maximizes expected utility
  - Note: an agent can be entirely rational (consistent with MEU) without ever representing or manipulating utilities and probabilities
  - E.g., a lookup table for perfect tictactoe, reflex vacuum cleaner

4

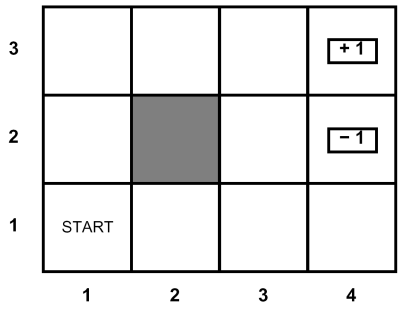
Recap MDPs and RL

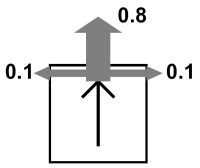
- Markov Decision Processes (MDPs)
  - Formalism (S, A, T, R, gamma)
  - Solution: policy pi which describes action for each state
  - Value Iteration (vs. Expectimax --- VI more efficient through dynamic programming)
  - Policy Evaluation and Policy Iteration
- Reinforcement Learning (don't know T and R)
  - Model-based Learning: estimate T and R first
  - Model-free Learning: learn without estimating T or R
    - Direct Evaluation [performs policy evaluation]
    - Temporal Difference Learning [performs policy evaluation]
    - Q-Learning [learns optimal state-action value function Q*]
    - Policy Search [learns optimal policy from subset of all policies]
  - Exploration
- Function approximation --- generalization

5

Markov Decision Processes

- An MDP is defined by:
  - A set of states s ∈ S
  - A set of actions a ∈ A
  - A transition function T(s,a,s')
    - Prob that a from s leads to s'
    - i.e., P(s' | s,a)
    - Also called the model
  - A reward function R(s, a, s')
    - Sometimes just R(s) or R(s')
  - A start state (or distribution)
  - Maybe a terminal state
- MDPs are a family of non-deterministic search problems
  - Reinforcement learning: MDPs where we don't know the transition or reward functions





6

What is Markov about MDPs?

- "Markov" generally means that given the present state, the future and the past are independent
- For Markov decision processes, "Markov" means:

$$P(S_{t+1} = s' | S_t = s_t, A_t = a_t, S_{t-1} = s_{t-1}, A_{t-1}, \ldots S_0 = s_0)$$

$$=$$

$$P(S_{t+1} = s' | S_t = s_t, A_t = a_t)$$

- Can make this happen by proper choice of state space

Value Iteration

- Idea:
  - $V_i^*(s)$: the expected discounted sum of rewards accumulated when starting from state s and acting optimally for a horizon of i time steps.
- Value iteration:
  - Start with $V_0^*(s) = 0$, which we know is right (why?)
  - Given V_i^*, calculate the values for all states for horizon i+1:

$$V_{i+1}^*(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_i^*(s') \right]$$

  - This is called a value update or Bellman update
  - Repeat until convergence
- Theorem: will converge to unique optimal values
  - Basic idea: approximations get refined towards optimal values
  - Policy may converge long before values do
  - At convergence, we have found the optimal value function V* for the discounted infinite horizon problem, which satisfies the Bellman equations: $\forall s \in S : \quad V^*(s) = \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^*(s') \right]$

8

Complete Procedure

- 1. Run value iteration (off-line)
  - This results in finding V*
- 2. Agent acts. At time t the agent is in state s_t and takes the action a_t:

$$\arg\max_a \sum_{s'} T(s_t, a, s')[R(s_t, a, s') + \gamma V^*(s')]$$

9

Policy Iteration

- Policy evaluation: with fixed current policy π, find values with simplified Bellman updates:
  - Iterate for i = 0, 1, 2, … until values converge

$$\forall s: \quad V_{i+1}^{\pi_k}(s) \leftarrow \sum_{s'} T(s, \pi_k(s), s') \left[R(s, \pi_k(s), s') + \gamma V_i^{\pi_k}(s')\right]$$

- Policy improvement: with fixed utilities, find the best action according to one-step look-ahead

$$\pi_{k+1}(s) = \arg\max_a \sum_{s'} T(s, a, s') \left[R(s, a, s') + \gamma V^{\pi_k}(s')\right]$$

- Will converge (policy will not change) and resulting policy optimal

10

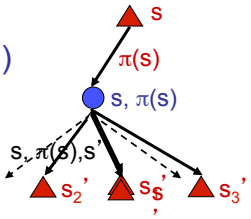
Sample-Based Policy Evaluation?

$$V_{i+1}^{\pi}(s) \leftarrow \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V_i^{\pi}(s')]$$

- Who needs T and R? Approximate the expectation with samples (drawn from T!)

$$sample_1 = R(s, \pi(s), s'_1) + \gamma V_i^{\pi}(s'_1)$$
$$sample_2 = R(s, \pi(s), s'_2) + \gamma V_i^{\pi}(s'_2)$$
$$\cdots$$
$$sample_k = R(s, \pi(s), s'_k) + \gamma V_i^{\pi}(s'_k)$$

$$V_{i+1}^{\pi}(s) \leftarrow \frac{1}{k} \sum_i sample_i$$

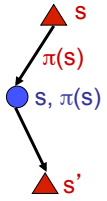


Almost! (i) Will only be in state s once and then land in s' hence have only one sample → have to keep all samples around? (ii) Where do we get value for s'?

11

Temporal-Difference Learning

- Big idea: learn from every experience!
  - Update V(s) each time we experience (s,a,s',r)
  - Likely s' will contribute updates more often
- Temporal difference learning
  - Policy still fixed!
  - Move values toward value of whatever successor occurs: running average!



Sample of V(s): $sample = R(s, \pi(s), s') + \gamma V^{\pi}(s')$

Update to V(s): $V^{\pi}(s) \leftarrow (1 - \alpha)V^{\pi}(s) + (\alpha)sample$

Same update: $V^{\pi}(s) \leftarrow V^{\pi}(s) + \alpha(sample - V^{\pi}(s))$

12

Exponential Moving Average

- Exponential moving average
  - Makes recent samples more important

$$\bar{x}_n = \frac{x_n + (1-\alpha)\cdot x_{n-1} + (1-\alpha)^2 \cdot x_{n-2} + \dots}{1 + (1-\alpha) + (1-\alpha)^2 + \dots}$$

  - Forgets about the past (distant past values were wrong anyway)
  - Easy to compute from the running average

$$\bar{x}_n = (1-\alpha)\cdot \bar{x}_{n-1} + \alpha \cdot x_n$$

- Decreasing learning rate can give converging averages

Detour: Q-Value Iteration

- Value iteration: find successive approx optimal values
  - Start with $V_0(s) = 0$, which we know is right (why?)
  - Given V_i, calculate the values for all states for depth i+1:

$$V_{i+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_i(s')\right]$$

- But Q-values are more useful!
  - Start with $Q_0(s,a) = 0$, which we know is right (why?)
  - Given Q_i, calculate the q-values for all q-states for depth i+1:

$$Q_{i+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q_i(s',a')\right]$$

Q-Learning

- Learn Q*(s,a) values
  - Receive a sample (s,a,s',r)
  - Consider your new sample estimate:

$$Q^*(s,a) = \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q^*(s',a') \right]$$

$$sample = R(s,a,s') + \gamma \max_{a'} Q(s',a')$$

  - Incorporate the new estimate into a running average:

$$Q(s,a) \leftarrow (1-\alpha) Q(s,a) + (\alpha) \, [sample]$$

- Amazing result: Q-learning converges to optimal policy
  - If you explore enough
  - If you make the learning rate small enough but not decrease it too quickly!
- Neat property: off-policy learning
  - learn optimal policy without following it

15

Exploration Functions

- Simplest: random actions (ε greedy)
  - Every time step, flip a coin
  - With probability ε, act randomly
  - With probability 1-ε, act according to current policy
  - Problems with random actions?
    - You do explore the space, but keep thrashing around once learning is done
    - One solution: lower ε over time
- Exploration functions
  - Explore areas whose badness is not (yet) established
  - Take a value estimate and a count, and returns an optimistic utility, e.g. $f(u,n) = u + k/n$ (exact form not important)

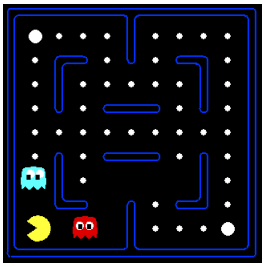
$$Q_{i+1}(s,a) \leftarrow (1-\alpha) Q_i(s,a) + \alpha \left(R(s,a,s') + \gamma \max_{a'} Q_i(s',a') \right)$$

now becomes:

$$Q_{i+1}(s,a) \leftarrow (1-\alpha) Q_i(s,a) + \alpha \left(R(s,a,s') + \gamma \max_{a'} f(Q_i(s',a'), N(s',a')) \right)$$

Feature-Based Representations

- Solution: describe a state using a vector of features
  - Features are functions from states to real numbers (often 0/1) that capture important properties of the state
  - Example features:
    - Distance to closest ghost
    - Distance to closest dot
    - Number of ghosts
    - 1 / (dist to dot)^2
    - Is Pacman in a tunnel? (0/1)
    - …… etc.
  - Can also describe a q-state (s, a) with features (e.g. action moves closer to food)



17

Linear Feature Functions

- Using a feature representation, we can write a q function (or value function) for any state using a few weights:

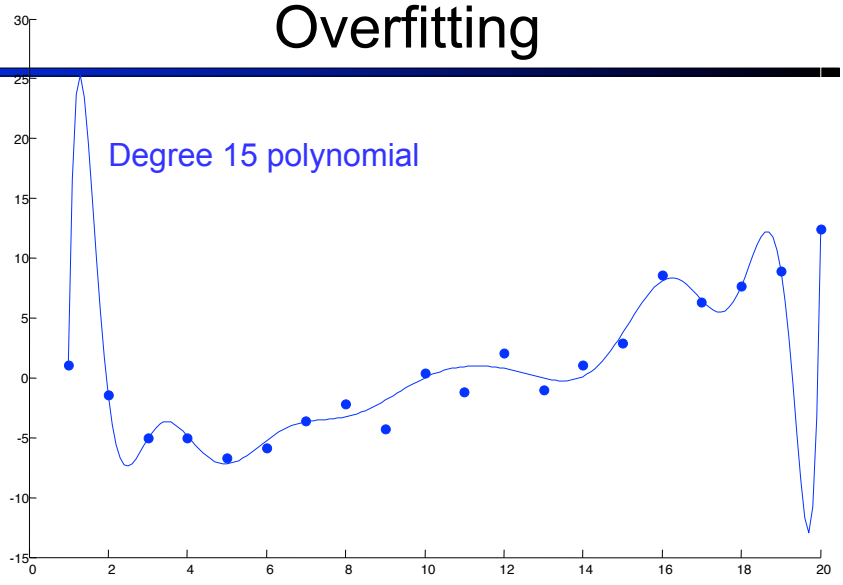
$$V(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

- Advantage: our experience is summed up in a few powerful numbers
- Disadvantage: states may share features but be very different in value!

18

Overfitting



19

Policy Search

- Problem: often the feature-based policies that work well aren't the ones that approximate V / Q best
- Solution: learn the policy that maximizes rewards rather than the value that predicts rewards
- This is the idea behind policy search, such as what controlled the upside-down helicopter
- Simplest policy search:
  - Start with an initial linear value function or Q-function
  - Nudge each feature weight up and down and see if your policy is better than before
- Problems:
  - How do we tell the policy got better?
  - Need to run many sample episodes!
  - If there are a lot of features, this can be impractical

20